Modelling Directional Hedge Funds
Mean, Variance and Correlation with Tracker Funds

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Abstract
Many hedge fund managers use some kind of systematic approach to actively trade the markets. Modelling the returns generated by these dynamic strategies requires allowing for market inefficiencies. The first two moments, expected value and variance are derived analytically for a general class of trading rules with potential forecasting ability. The correlation coefficient between the active program and a tracker fund is subsequently derived allowing for mean-variance allocation. The use of theoretical formulae improves the accuracy of the parameter's estimates. Furthermore, this permits the construction of ex-ante optimal portfolios. When the underlying markets shows a positive drift and the goal is to maximize the return to risk ratio on the investment, a portfolio including both a tracker fund and a long/short strategy will be superior to both investments considered separately. The forecasting model used to time the derivatives markets will have to take into account the tracker fund it is associated to if the goal is to provide the highest return to risk ratio. Finally, we provide an empirical application in the currency markets. We show how two popular overlay strategies can be added to a benchmark and how theoretical modelling can be used.

Alternative investment has grown in popularity over the past few years. The main reason being the decline in stock markets and the lack of correlation between hedge fund returns and major indices. Still, theoretical work on hedge funds is at its infancy. The academic research can be roughly split into two areas. The first category studies the historical track records generated by these managers and attempts to replicate their performance using popular strategies. The fact is that many hedge fund managers base their trading decisions on models. They often receive the label of "systematic" managers. Therefore it is not unreasonable to think that their returns can be replicated or at least explained by basic market factors. These studies are informative because they provide a better understanding of the trading process followed by hedge fund managers. In the Futures markets, trend-following indicators are especially used. However the range of explanatory factors is much wider in other markets and the reader is referred to Schneeweis and Spurgin (1998), Fung and Hsieh (2001), Mitchell and Pulvino (2001) and Martin (2001) for an in-depth analysis of hedge fund strategies. The second category of research concentrates on establishing the return distributions of dynamic strategies. Directional trading rules are typical examples of techniques affecting the distribution of return (Acar and Satchell, 1998). Extension of this work and its relevancy to hedge

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All errors remain mine.

fund management has been investigated by Lundin and Satchell (2000). Generalisation to active fund management and relative returns has been recently formulated in Hwang and Satchell (2001). Most of these studies use the assumption of no forecasting ability to achieve analytical developments. These results are useful because they provide the statistical means to build random walk tests, or value at risk estimators. Our goal is here to combine both parties, that is to reconcile empirical observations with analytical formulae. Modelling the returns generated by directional strategies, especially their expected value, requires allowing for market inefficiencies. More specifically, Section 1 establishes the first two moments of the returns generated by the directional predictor allowing for forecasting ability. Our theoretical work draws from Acar (1998) but generalizes the findings to a much broader class of active strategies including long/short, short only, long only and asymmetrical positioning. Section 2 studies the effect of combining a tracker fund with a hedge fund. Section 3 performs some Monte-Carlo simulations to compare the efficiency of estimating mean/variance and correlation of hedge fund returns either directly or using our formulae. Section 4 discusses the issue of optimal allocation when the goal is to maximize the return to risk ratio of the portfolio. Finally, Section 5 provides an empirical application in the currency markets. We show how two popular overlay strategies can be added to a benchmark and how theoretical modelling can be used.

1) **Mean and Variance of Directional Strategies**

Let us consider a money manager trading an underlying asset whose returns are denoted X. To generate his position, the trader uses a forecasting technique to predict the sign of the forthcoming returns. The forecast F decides if the asset is to be bought or sold. We study here the general trading process where units in quantity a are hold when the forecast is positive and units in quantity b are hold when the forecast is negative. A rationale rule consists in buying the asset (a≥0) when the forecast is positive and selling the asset (b≤0) when the forecast is negative. However it could be that the strategy is constrained to be long only and that a>b≥0. To consider the most general case, we do not put any restrictions on the parameters a and b and the formulae developed in this section are also valid for any values of a and b including a<b or b=0. This is a significant generalization of Acar (1998) and Skouras (2001) who only consider symmetrical long/short strategies of the form $a = -b = 1$. The returns generated by the forecasting rule are denoted H. That is:

$$H = \begin{cases} a\,X & \text{if} \quad F > 0 \\ b\,X & \text{if} \quad F < 0 \end{cases}{}^{2} \qquad [1]$$

The forecasting technique could be a technical indicator, an auto-regressive predictor or an exogenous variable. Two examples in the currency markets would be moving averages and interest rate differential (See section 5 for further details). Here, we assume that the joint distribution of the underlying returns X and the forecast F is a bivariate normal distribution denoted by:

$$\begin{bmatrix} X \\ F \end{bmatrix} \sim N(\begin{bmatrix} \mu_x \\ \mu_f \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \rho_{xf}\sigma_x\sigma_f \\ \rho_{xf}\sigma_x\sigma_f & \sigma_f^2 \end{bmatrix})$$

The random walk hypothesis would imply that there cannot be any correlation between the forecast and the forthcoming market returns. That is $\rho_{xf} = 0$. Appendix 1 demonstrates that under the assumption that the underlying market and the forecast follow a bivariate distribution, the directional strategy has for first two moments:

$$E(H) = \mu_x [a\, \Phi(\frac{\mu_f}{\sigma_f}) + b\, \Phi(-\frac{\mu_f}{\sigma_f})] + \sigma_x \frac{(a-b)}{\sqrt{2\pi}} \rho_{xf} \exp(-0.5 \frac{\mu_f^2}{\sigma_f^2}) \qquad [2]$$

[2] The case F=0 is considered of zero probability.

$$E(H^2) = \mu_x^2[a^2\ \Phi(\frac{\mu_f}{\sigma_f}) + b^2\ \Phi(-\frac{\mu_f}{\sigma_f})] + 2\mu_x\sigma_x\ \frac{(a^2 - b^2)}{\sqrt{2\pi}}\rho_{xf}\ \exp(-0.5\frac{\mu_f^2}{\sigma_f^2})$$
$$+\sigma_x^2[a^2(\frac{\rho_{xf}^2}{\sqrt{2\pi}}(-\frac{\mu_f}{\sigma_f})\exp(-0.5\frac{\mu_f^2}{\sigma_f^2}) + \Phi(\frac{\mu_f}{\sigma_f})) + b^2(\frac{\rho_{xf}^2}{\sqrt{2\pi}}(\frac{\mu_f}{\sigma_f})\exp(-0.5\frac{\mu_f^2}{\sigma_f^2}) + \Phi(-\frac{\mu_f}{\sigma_f}))] \quad [3]$$

where Φ is the cumulative function of a normal distribution N(0,1).
The variance is simply given by the relationship $Var(H) = E(H^2) - (E(H))^2$ and the standard deviation $Std(H) = \sqrt{Var(H)}$.
Formula [2] may explain some of the empirical results observed by Schneeweis and Spurgin (1998). They find that Commodity Trading Advisors (CTAs) returns are positively correlated to factors such as market trends and currency movements, while hedge fund and mutual fund returns are best explained by the return to a buy and hold strategy in the markets the fund invests in. On the one hand, CTA usually trade the futures markets, which over long period of time exhibit small drift. The profit generated by active programs has got to come therefore from the second block in equation [2], the product of the market volatility with the correlation coefficient between forecast and future moves. On the other hand, in markets exhibiting strong drift, the first term in equation [2] is more likely to be the dominant factor.
Higher moments of the hedge fund returns also accept exact analytical formulae. They are not reproduced here for length purpose but the interested reader is referred to Kotz et al (2000: p315) or Chou and Owen (1984) who develop the explicit formulas for the cumulants of a variable of the bivariate normal distribution when the other variable is truncated below. These values are useful for obtaining via the Cornish Fisher expansion, approximation to percentage points of the hedge fund returns. An inspection of the cumulants (Kotz, et al; 2000: p315) shows that the correlation coefficient between forecast and market significantly affects the mean of the hedge fund return, but its effect on higher moment decreases rapidly. This effect is particularly pronounced in Finance since the correlation coefficient is very unlikely to exceed 0.3. The amounts of skewness and kurtosis will be mostly a function of the trading units, the parameters a and b rather than the correlation coefficient between forecast and market. The less symmetrical the strategy, the bigger the magnitude of the skewness and kurtosis coefficients. In particular, "polar" strategies (a=1, b=0 or a=0, b=-1) are likely to generate returns deviating significantly from a normal distribution simply because of the abnormally large number of zero observations.

2) **Correlation with tracker fund**

So far we have only considered systematic programs as an asset class in isolation of any other investment. In practise, directional hedge funds do not purely and simply replace traditional investments but complement them to form a fully diversified portfolio. Indeed, an interesting approach already adopted by a few large institutions, is to construct a portfolio including both a tracker fund and a hedge fund. The inclusion of hedge funds in a portfolio can potentially result in better risk-return tradeoffs due to the low correlation between hedge fund returns and the returns on the traditional asset classes like equities or bonds (Fung and Hsieh, 1997). However to realize an effective allocation between a tracker fund and a hedge fund, a crucial parameter is needed, the correlation coefficient between the two investments. As pointed out in Schneeweis and Spurgin (2000), it is important to realize that while hedge fund managers do not emphasize benchmark tracking this does not mean that their entire return is based solely on manager skill or is independent of the movement of underlying stock, bond or currency markets. The correlation coefficient with the tracker is clearly going to be negative for short only fund and positive for long only funds, whereas it may be closer to zero for market neutral hedge funds. It is notoriously difficult to estimate the correlation coefficient between hedge funds and benchmarks from historical returns. Previous research has identified a tendency for hedge fund and managed futures strategies to exhibit non-constant correlation with the US equity market. Schneeweis and Spurgin (1998) observed that many market-neutral strategies exhibit higher correlation with stock and bond benchmarks during the market declines, and lower correlation during rallies. Schneeweis and Spurgin (2001) outlines a simple econometric method of estimating changes in correlation from historical returns. Indeed, a precise quantification of the correlation coefficient is key for use in the asset allocation decision. A complementary approach is to establish the theoretical correlation coefficient. This can be achieved when the hedge fund trading process is modelled by equation [1]. We simply need to incorporate the tracker fund in our modelling process. Here, we explicitly assume that the joint distribution of the underlying returns X, the forecast F and the tracker fund T is a trivariate normal distribution denoted by:

$$\begin{bmatrix} X \\ F \\ T \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_x \\ \mu_f \\ \mu_t \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \rho_{xf}\sigma_x\sigma_f & \rho_{xt}\sigma_x\sigma_t \\ \rho_{xf}\sigma_x\sigma_f & \sigma_f^2 & \rho_{ft}\sigma_f\sigma_t \\ \rho_{xt}\sigma_x\sigma_t & \rho_{ft}\sigma_f\sigma_t & \sigma_t^2 \end{bmatrix}\right)$$

The hedge fund returns H and the tracker fund T will not follow a bivariate normal distribution. However for mean variance allocation purposes, it is still important to assess the correlation coefficient between the tracker fund and the hedge fund Corr(T,H) which is the only parameter missing at this stage. The basic idea is that the covariance between the two variables can be deduced from the variance of the portfolio including both the Tracker fund and the Hedge fund. Let's note the portfolio P=T+H, $\delta_f = \mu_f / \sigma_f$ the return to risk ratio of the forecast and

$$g[\mu_x, \sigma_x^2, \delta_f, \rho_{xf}] = \mu_x^2 \Phi(\delta_f) + 2\mu_x\sigma_x \frac{\rho_{xf}}{\sqrt{2\pi}} \exp(-\frac{\delta_f^2}{2}) + \sigma_x^2 (\frac{-\rho_{xf}^2}{\sqrt{2\pi}} \delta_f \exp(-\frac{\delta_f^2}{2}) + \Phi(\delta_f)).$$

Appendix 2 shows that:

$$E(P) = \mu_t + E(H)$$

$$E(P^2) = g[\mu_t + a\mu_x, \sigma_t^2 + a^2\sigma_x^2 + 2a\rho_{xt}\sigma_t\sigma_x, \delta_f, \frac{\rho_{ft}\sigma_t + a\rho_{xf}\sigma_x}{\sqrt{\sigma_t^2 + a^2\sigma_x^2 + 2a\rho_{xt}}}]$$

$$+ g[\mu_t + b\mu_x, \sigma_t^2 + b^2\sigma_x^2 + 2b\rho_{xt}\sigma_t\sigma_x, -\delta_f, -(\frac{\rho_{ft}\sigma_t + b\rho_{xf}\sigma_x}{\sqrt{\sigma_t^2 + b^2\sigma_x^2 + 2b\rho_{xt}}})]$$

$$Var(P) = E(P^2) - (E(P))^2$$

and $$\mathrm{Corr}(T,H) = \frac{Var(P) - \sigma_t^2 - Var(H)}{2\,\sigma_t\sqrt{Var(H)}} \quad [4]$$

It can be shown by developing [4] that the correlation coefficient between the tracker and the hedge fund will be equal to zero irrespective of the values of ρ_{xt}, ρ_{ft}, and ρ_{xf} when the following three conditions are met. Firstly, the underlying actively traded market X has zero mean $\mu_x = 0$. Secondly, the forecast has zero mean $\mu_f = \delta_f = 0$. Lastly, the trading process is long/short in equal quantities $b = -a$. This implies that we can think of combining a cash index fund with a derivatives program trading exclusively the index futures contracts. If the mean of the index futures contracts is zero as well as the mean of the forecast, then the correlation coefficient between the cash index and the short/long futures program will be zero. This intuitive result may not hold as soon as the market drift is different from zero or the strategy is not long/short in equal quantity ($b \neq -a$) and/or probability ($\mu_f \neq 0$).

3) **Parameters estimation**

The benefit of a theoretical formula to estimate the hedge fund mean, variance and correlation with other asset classes is primarily in the improved understanding of the market conditions required for out-performance. Sensitivity analysis is rendered possible. Here we want to make a further point that by using theoretical formulae better estimates of mean, variance and correlation may be achieved. If we think of short-only hedge funds, active positioning may be infrequent and if we only have access to the hedge funds returns we may have very few observations. On the other hand, studying the forecasting strategy and its relation to the underlying market may be more valuable since this is similar to observing a distribution before truncation. Let's take the example of a normal random walk with no drift for all the variables, the underlying market, forecast and tracker fund. We further assume zero correlations between all the variables:

$$\begin{bmatrix} X \\ F \\ T \end{bmatrix} \sim N(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix})$$

Of course the hedge fund will have zero expected value $E(H) = 0$ and variance $Var(H) = 0.5\,(a^2 + b^2)$. An informative experimentation is to use Monte-Carlo simulation to compare the estimates of the hedge fund moments either provided directly using the empirical observations for H or implied by the joint distribution of [X, F, T]. Measurement errors will affect both methodologies but the use of theoretical formulae may exacerbate or decrease those. Since very few hedge funds have more than ten years track records of monthly returns, series of 36, 60 and 120 observations were simulated ten thousand times for two sets of strategies, long/short with equal quantity $a = -b = 1$ and short only strategies $a = 0, b = -1$.

For long/short strategies (Table 1), the use of analytical formulae only marginally improves the estimations of the mean and standard deviation. However the correlation coefficient between the tracker and the hedge fund converges towards its expected value of zero much more quickly as seen by a smaller range maximum minus minimum. For short only strategies (Table 2), the estimations implied from the joint distribution of X, F and T are more accurate that row observations for both the standard deviation of hedge fund returns and its correlation with the tracker fund. However the magnitude of improvement is reduced.

Table 1: Long/short strategies

	Observed from (H,T)			Implied from (X,F,T)		
36 months	E(H)	Std(H)	Corr(T,H)	E(H)	Std(H)	Corr(T,H)
Average	0.0027	0.9923	0.0007	0.0007	0.9972	0.0001
Standard Deviation	0.1664	0.1197	0.1678	0.1339	0.1194	0.0312
Minimum	-0.7072	0.5525	-0.5699	-0.5761	0.5643	-0.1636
Maximum	0.6833	1.4890	0.5807	0.4775	1.4972	0.1747
60 months	E(H)	Std(H)	Corr(T,H)	E(H)	Std(H)	Corr(T,H)
Average	-0.0013	0.9956	0.0001	-0.0010	0.9984	0.0000
Standard Deviation	0.1275	0.0905	0.1307	0.1030	0.0904	0.0186
Minimum	-0.4794	0.5962	-0.4944	-0.4183	0.5973	-0.1205
Maximum	0.4792	1.3347	0.4421	0.3883	1.3450	0.1290
120 months	E(H)	Std(H)	Corr(T,H)	E(H)	Std(H)	Corr(T,H)
Average	-0.0003	0.9987	-0.0004	-0.0006	1.0002	0.0000
Standard Deviation	0.0911	0.0648	0.0910	0.0732	0.0647	0.0093
Minimum	-0.3492	0.7748	-0.3506	-0.2743	0.7757	-0.0511
Maximum	0.3395	1.2595	0.3402	0.3029	1.2602	0.0486

Table 2: Short-only strategies

	Observed from (H,T)			Implied from (X,F,T)		
36 months	E(H)	Std(H)	Corr(T,H)	E(H)	Std(H)	Corr(T,H)
Average	0.0007	0.6951	-0.0011	0.0009	0.7019	-0.0017
Standard Deviation	0.1169	0.1327	0.1692	0.1068	0.0960	0.1193
Minimum	-0.3855	0.2312	-0.5991	-0.3711	0.3751	-0.4396
Maximum	0.4075	1.1755	0.5997	0.4162	1.1104	0.4271
60 months	E(H)	Std(H)	Corr(T,H)	E(H)	Std(H)	Corr(T,H)
Average	-0.0004	0.6999	0.0006	-0.0001	0.7043	0.0005
Standard Deviation	0.0911	0.1018	0.1287	0.0821	0.0737	0.0914
Minimum	-0.3597	0.3576	-0.4539	-0.3566	0.4586	-0.3316
Maximum	0.3403	1.0913	0.4897	0.3241	1.0041	0.3352
120 months	E(H)	Std(H)	Corr(T,H)	E(H)	Std(H)	Corr(T,H)
Average	-0.0003	0.7048	-0.0005	-0.0004	0.7066	-0.0007
Standard Deviation	0.0648	0.0716	0.0918	0.0589	0.0521	0.0656
Minimum	-0.3161	0.3959	-0.3215	-0.2673	0.5124	-0.2550
Maximum	0.2680	0.9683	0.3710	0.2345	0.9344	0.2441

4) **Optimal allocation**

Let's consider an investor holding a tracker fund and willing to add a hedge fund to its portfolio. His resulting portfolio will be $P = T + w\ H$. The weight w to be given to the hedge fund depends on the investor utility function. This section considers that the investor is willing to maximise his return divided standard deviation ratio $SR(P) = E(P)/\sqrt{Var(P)}$, the optimal weight w* can be derived for given directional strategies using the steps similar to Elton et al (1987). The results of the previous section can be used to establish $\mu_h = E(H)$, $\sigma_h = \sqrt{Var(H)}$, and $\rho_{th} = Corr(T,H)$.

Under the assumptions that:

$$\left(\frac{\mu_h}{\sigma_h}\right) > \left(\frac{\mu_t}{\sigma_t}\right)\rho_{th} \text{ and } \left(\frac{\mu_t}{\sigma_t}\right) > \left(\frac{\mu_h}{\sigma_h}\right)\rho_{th}, \quad w^* = \frac{\left(\frac{\mu_h}{\sigma_h}\right) - \left(\frac{\mu_t}{\sigma_t}\right)\rho_{th}}{\left(\frac{\mu_t}{\sigma_t}\right) - \left(\frac{\mu_h}{\sigma_h}\right)\rho_{th}} \frac{\sigma_t}{\sigma_h} \qquad [5]$$

The optimal weight may be larger than one, but since the allocation is made to a hedge fund, this would mean increasing the leverage used by the hedge fund manager. This is often possible given that the latter trades the derivatives markets.

An interesting question is to know if an actively managed fund is preferable to the combination of a tracker fund and a hedge fund or vice-versa. Indeed, it could be claimed that an actively managed fund can always be decomposed as the sum of a tracker fund and a hedge fund. The benefit generated by such decomposition is that it allows the asset allocator and/or investor to adjust the weightings between passive and active management. In the most general case, the number of degree of freedom to build the trading rule is too large to bring a definite conclusion. However some comparison can be investigated analytically if we restrict ourselves to long/short strategies in equal quantity $a = -b = 1$. Let us consider an index returns X with positive mean μ_x, and standard deviation σ_x. We assume that there exists a futures market, X^* which differs from the underlying market only by the long-term drift. We do not suppose the existence of any risk premium. That is $X^* = X - \mu_x$. The question is which of the two portfolios produces the highest return to risk ratio, an actively managed fund on the index itself or a portfolio including the index and an actively managed index futures program. Let's establish the corresponding maximum returns to risk ratios for both investments.

Optimal long/short fund on the index itself X

Our long/short strategy is defined by:

$H = \begin{Bmatrix} X & \text{if} & F > 0 \\ -X & \text{if} & F < 0 \end{Bmatrix}$ where F is a forecast used to predict the index market X

It is worthwhile noting that in this case $E(H^2) = E(X^2) = \mu_x^2 + \sigma_x^2$ and $Var(H) = \mu_x^2 + \sigma_x^2 - E(H)^2$. As a consequence, the forecast that maximises returns will also minimise variance[3]. Acar (1998) shows that such a forecast needs to maximise ρ_{xf} and satisfies the equation $\frac{\mu_f}{\sigma_f} = \frac{\mu_x}{\sigma_x \rho_{xf}}$. If we denote H_{max} the returns derived by the optimal predictor, we obtain from equation [2]:

$$E(H_{max}) = \mu_x [1 - 2\,\Phi(-\frac{\mu_x}{\sigma_x \rho_{xf}})] + \sigma_x \sqrt{\frac{2}{\pi}} \rho_{xf} \exp(-0.5 \frac{\mu_x^2}{\sigma_x^2 \rho_{xf}^2})$$

$$SR(H_{max}) = \frac{E(H_{max})}{\sqrt{\mu_x^2 + \sigma_x^2 - E(H_{max})^2}}$$

It can be noted that $SR(H)_{max}$ is an only function of the return to risk ratio $\delta_x = \mu_x / \sigma_x$ and ρ_{xf}.

$$SR(H_{max}) = \frac{\delta_x [1 - 2\,\Phi(-\frac{\delta_x}{\rho_{xf}})] + \sqrt{\frac{2}{\pi}} \rho_{xf} \exp(-0.5 \frac{\delta_x^2}{\rho_{xf}^2})}{\sqrt{\delta_x^2 + 1 - (\delta_x [1 - 2\,\Phi(-\frac{\delta_x}{\rho_{xf}})] + \sqrt{\frac{2}{\pi}} \rho_{xf} \exp(-0.5 \frac{\delta_x^2}{\rho_{xf}^2}))^2}}$$

Figure 1 plots the maximum return to risk ratio generated by the directional forecast as a function of the return to risk ratio of the underlying market and the correlation between the forecast and the cash index market. When there is little correlation, the market timing ability is low and the optimal strategy is close from Buy and Hold. However the presence of positive correlation between forecast and the underlying market permits the directional hedge fund to exhibit a superior return to risk ratio. It is worthwhile noting that the excess returns are larger, the lower the drift in the cash market. In other words, fund managers have an incentive to trade a zero drift market when they are evaluated in terms of relative performance against a Buy and Hold benchmark.

[3] Strictly speaking the forecast the forecast maximizing the absolute value of the expected return will minimise variance. However if the predictor generates a loss, it is always possible to establish the reverse strategy generating the maximum gain.

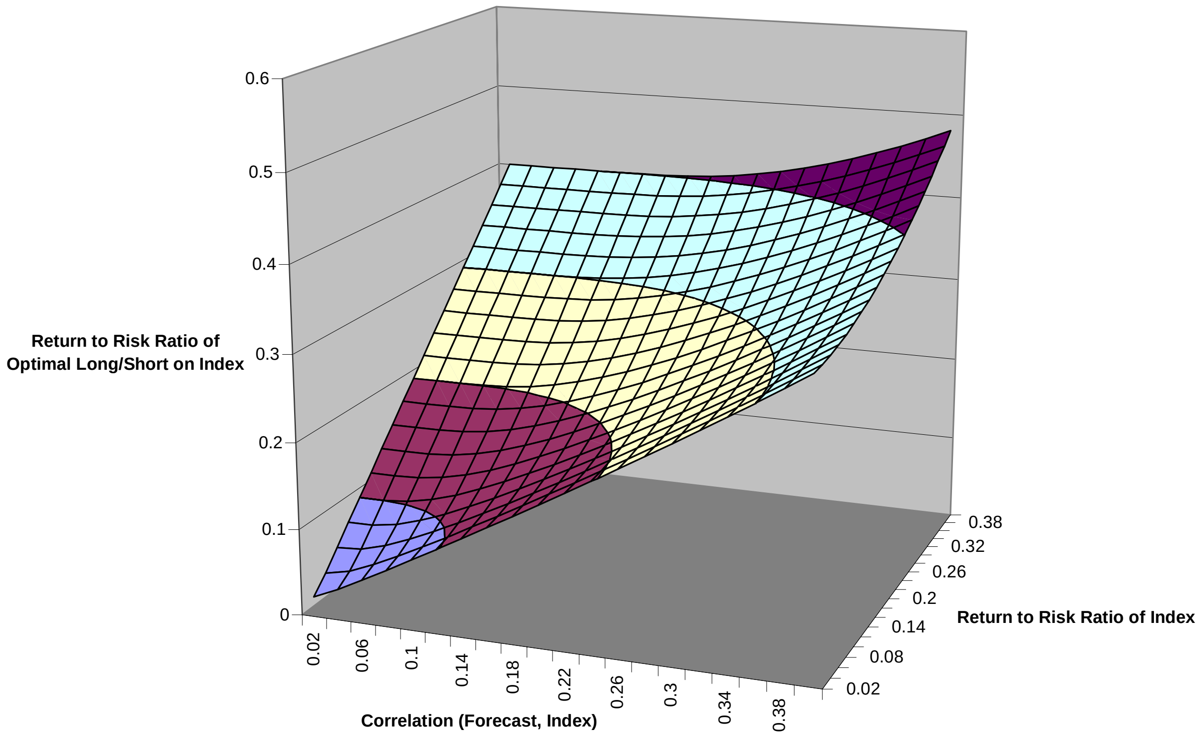

Figure 1: Maximum Return to Risk ratio of Optimal long/short strategy

Optimal allocation between an index and the maximum return long/short futures fund

Before investigating the general portfolio of an index fund with a managed futures program, we consider first the case of a separate futures program. This means that the manager seeks to maximise first the return on the futures contract and does not take into consideration the fact that his trading may not be optimal when combined to an index fund. This is very often the case in practise. Hedge fund managers design vehicles, which have for goals to maximise risk-adjusted returns on a stand-alone basis. No hedge fund manager will modify their strategies as a function of the index fund they may be added to.

$$P = X + w\,H^* \text{ and } H^* = \begin{cases} X^* & \text{if } F > 0 \\ -X^* & \text{if } F < 0 \end{cases}$$

Where F is the forecast used to predict the Futures market X^*.

We have just seen that maximising the returns on the long/short futures fund requires maximising the correlation between the forecast and the futures markets (or the index itself) ρ_{xf} and satisfies the equation $\delta_f = \mu_f / \sigma_f = 0$. If we denote H^*_{max} the returns derived by the optimal predictor on the dritfless futures markets, we obtain from equation [2]:

$$E(H^*_{max}) = \sigma_x \sqrt{\frac{2}{\pi}}\rho_{xf} \text{ and } Var(H^*_{max}) = \sigma_x^2 (1 - \frac{2}{\pi}\rho_{xf}^2)$$

We also know from Section 2 that for long/short strategies in a driftless market $Cov(H^*_{max}, X^*) = Cov(H^*_{max}, X) = 0$.

The second step is to allocate the optimal weight w between tracker and hedge fund given by [5].

$$\tilde{w} = \frac{\sigma_x}{\mu_x} \frac{\sqrt{\frac{2}{\pi}}\rho_{xf}}{(1 - \frac{2}{\pi}\rho_{xf}^2)}$$

Simple algebra provided in Appendix 3 shows that:

$$SR(P) = \frac{\delta_x^2 + \frac{2}{\pi}\rho_{xf}^2}{\sqrt{1-\frac{2}{\pi}\rho_{xf}^2}\sqrt{\delta_x^2 + \frac{2}{\pi}\rho_{xf}^2(1-\delta_x^2)}}$$

It can be noted once again that $SR(P)$ is again an only function of the return to risk ratio δ_x and ρ_{xf}. Figure 2 plots the return to risk ratio generated by the optimal combination of the tracker and the directional forecast maximizing return on the derivatives, as a function of the return to risk ratio of the underlying market and the correlation between the forecast and the forthcoming market returns.

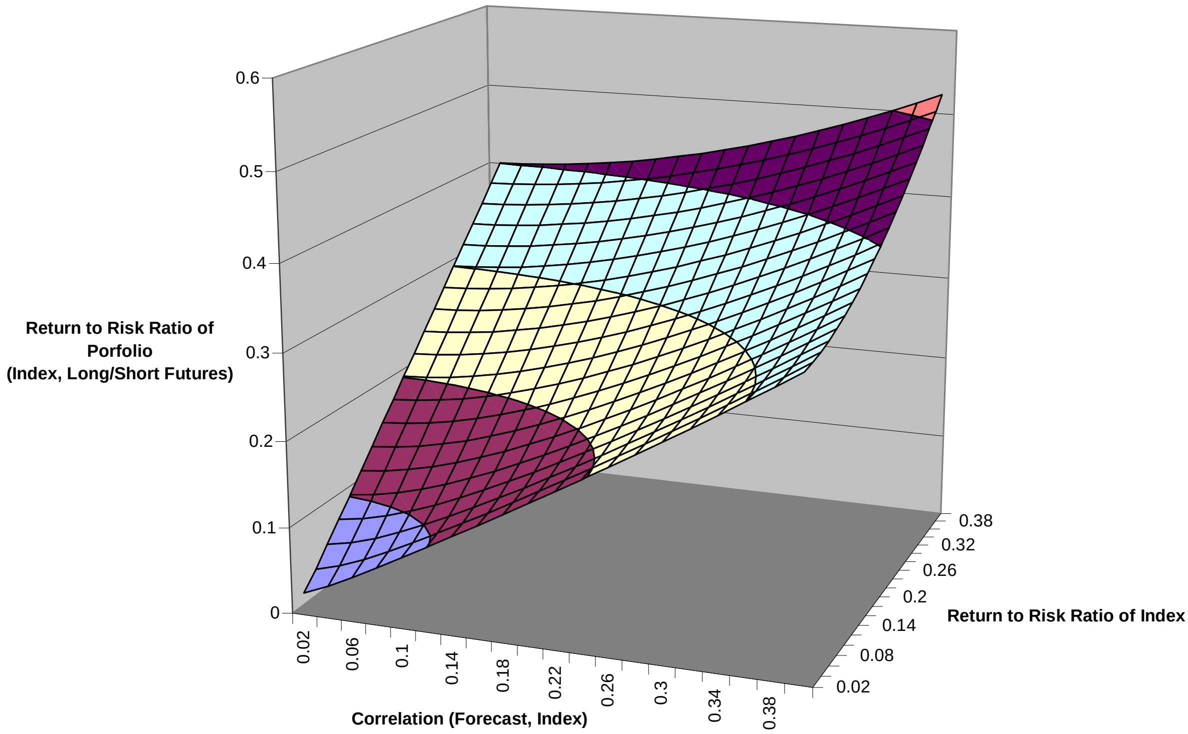

Figure 2: Maximum Return to Risk ratio of optimal Portfolio including the Index and the long/short strategy maximising returns on the Futures markets

Figure 3 specifies the weight actually given to the futures fund. Small values of the index fund tend to generate extremely large values on the derivatives position. They are not therefore being represented. As anticipated, the bigger the forecasting ability as measured by the correlation coefficient the bigger the allocation to the futures fund. This effect increases as the mean of the index fund decreases.

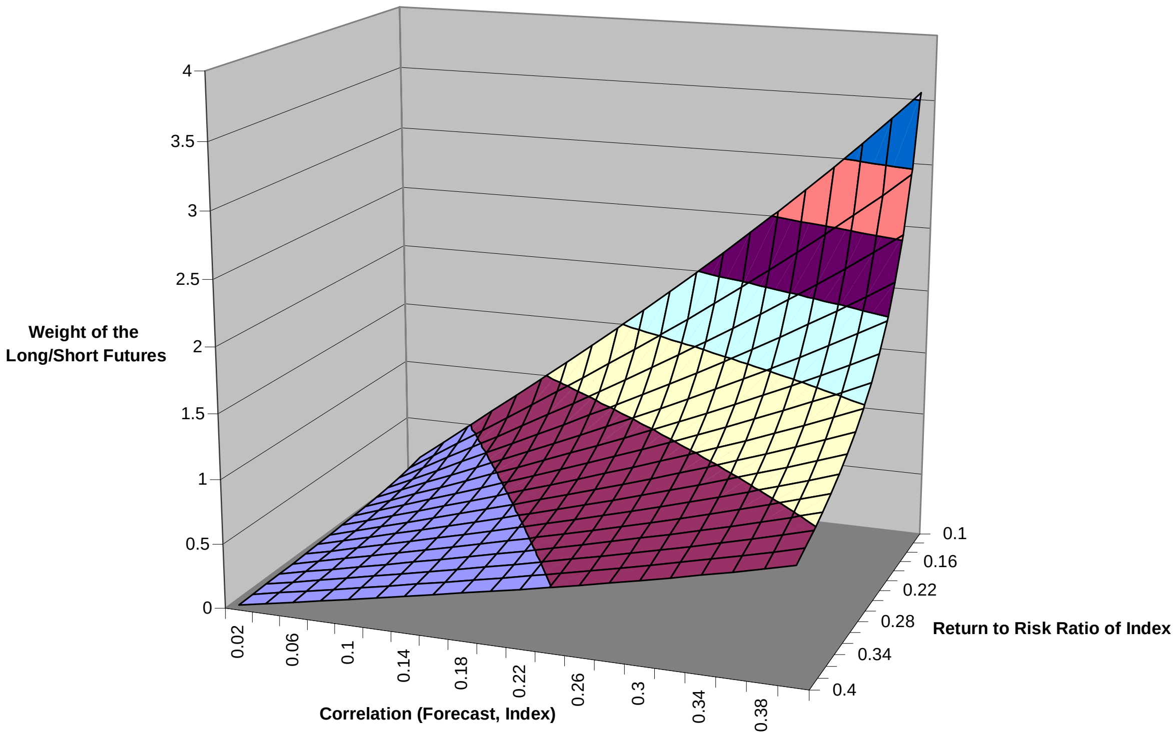

Figure 3: Optimal weight on the long/short strategy maximising returns on the Futures markets within a Portfolio including the Index

A close examination of the mathematical formulae would suggest that for $\rho_{xf} > 0$, $\delta_x > 0$, $SR(P) > SR(H_{max})$. The differences are the biggest when the drift and correlation coefficient are the highest. The maximum improvement is in the order of 0.05. This has important consequences since this suggests that going long/ short in equal quantities (but not necessarily in equal probability) in a market with drift will be sub-optimal to combining a passive position and an active long/short program in the driftless market.

Optimal allocation between an index and a long/short futures fund

The reader should bear in mind that the previous portfolio had been constructed to first maximise the returns on the futures market. The derivatives program was then given the weight maximizing the return to risk ratio of a portfolio also including the cash market. This two separate steps process does not guarantee the reach of a global maximum. A joint study of the forecasting strategy and weight is necessary. Let's establish ratio $SR(P) = E(P)/\sqrt{Var(P)}$ in its most general form.

$$P = X + w\,H^* \text{ and } H^* = \begin{Bmatrix} X^* & \text{if} & F > 0 \\ -X^* & \text{if} & F < 0 \end{Bmatrix}$$

$$E(P) = \mu_x + w\,E(H^*),\ Var(P) = \sigma_x^2 + w^2\ Var(H^*) + 2\,w\,Cov(H^*, X)$$

If we note $\delta_f = \mu_f / \sigma_f$, we know using [2], [3] and [4] that

$$E(H^*) = \sigma_x \sqrt{\frac{2}{\pi}} \rho_{xf} \exp(-0.5\delta_f^2),\ E(H^{*2}) = \sigma_x^2$$

$$Cov(H^*, X) = \sigma_x^2(-2\delta_f \sqrt{\frac{2}{\pi}} \rho_{xf}^2 \exp(-0.5\ \delta_f^2) + 4(\Phi(\delta_f) - 0.5))$$

The function $Cov(H^*, X)$ is an increasing of δ_f irrespective of the value of the correlation ρ_{xf}. This is negative for $\delta_f < 0$, equal to zero when $\delta_f = 0$ and positive when $\delta_f > 0$. Let's only consider the case where both the index and the futures fund have positive expected value. That is $\delta_x > 0$ and $\rho_{xf} > 0$. The case $\delta_f > 0$ cannot lead to a maximum return to risk ratio for the all portfolio. Having $\delta_f > 0$ generates lower expected value on the futures fund than having $\delta_f = 0$ and higher correlation with the index. On the other hand, $\delta_f < 0$ may lead to a global maximum. Having $\delta_f < 0$ again generates lower expected value on the futures fund than having $\delta_f = 0$. However this time, the correlation with the index is negative and it could be that the risk reduction compensates the decrease in returns. Injecting these results in equation [5] suggests that the optimal weight will be an only function of δ_x, δ_f and ρ_{xf}. Figure 3 charts one example of $SR(P) = E(P)/\sqrt{Var(P)}$ as a function of δ_f for a given value of $\delta_x = 0.1$ and $\rho_{xf} = 0.1$. The maximum return to risk ratio of 0.19 is obtained for δ_f equal to approximately –1.36. This is a considerable improvement to the previous allocation that forced $\delta_f = 0$ and could only achieve a ratio of 0.13. Therefore the benefits of combining tracker and directional hedge funds will be greater if the directional strategy applied to the derivatives markets takes into account the tracker it is combined to[4]. This raises an important issue since in practise it is unlikely that the hedge fund manager will be willing to refine his/her strategy as a function of the asset it is associated to.

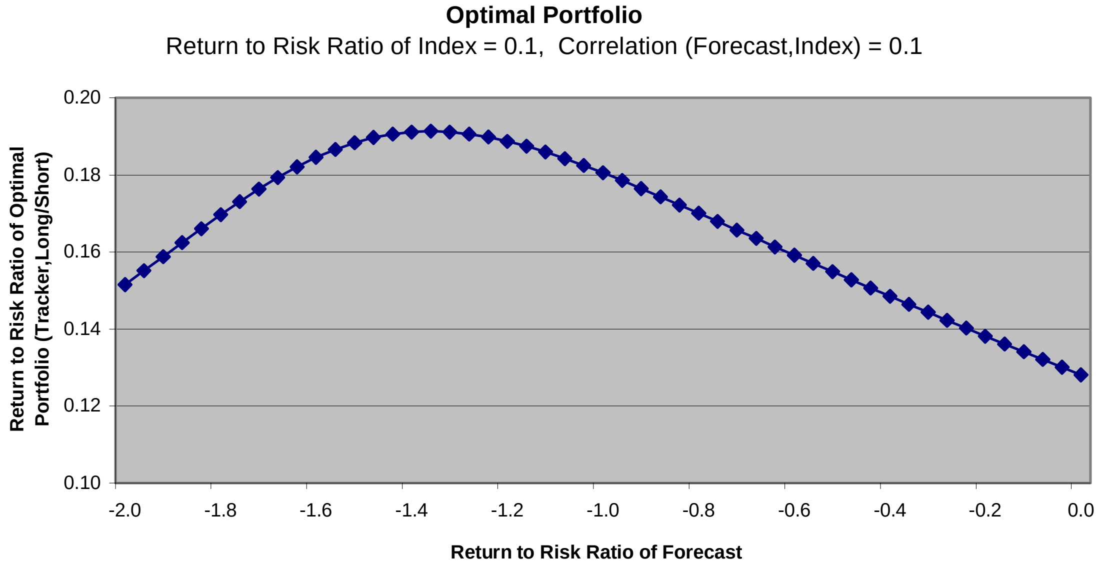

Figure 3: Maximum Return to Risk ratio of Optimal Portfolio including the Index and a long/short strategy as a function of the return to risk of the forecast when the return to risk ratio of the index = 0.1 and the correlation (forecast, index) = 0.1

5) __An empirical application to the currency markets__

This section investigates the explanatory power of our theoretical modelling. Attention is restricted to the mean, standard deviation of the directional hedge fund as well as its correlation with the tracker fund. For illustrative purposes, we consider a hedge fund trading currency. The underlying markets X is therefore the spot currency appreciation plus the carry in percentage terms over a month. The first forecasting strategy AR1 is based on an autoregressive model of length one. This

[4] The improvement will only affect investors exhibiting a quadratic utility function. Different values of δ_f will generate different skewness and kurtosis coefficients.

trend-following strategy simply says buy the currency after a monthly appreciation and sell it after a depreciation. The second strategy FRB is the forward rate bias, formally defined as buy the currency pair if the forthcoming monthly carry is positive and sell it if this negative. We express the forecasting strategy in terms of carry rather than hedged returns (the opposite) such that the correlation between forecast and forthcoming returns is sought to be positive. Tables 1 and 2 provide summary statistics respectively on the underlying currency markets X and the forecasting strategies F.
Both strategies have been studied by academics and applied by currency fund managers over the past twenty years. Strange (2001) gives an example of how these trading rules can be applied to hedge the currency exposure of international portfolios. In our first example, the benchmark will be a 50% hedged bond benchmark. We can then add a long/short currency overlay on 50% of the position $a = -b = 0.5$, denoted 0.5 l/s in Table 3. The second benchmark will be an unhedged bond benchmark, to which we add a short only currency overlay $a = 0$, $b = -1$. Our last benchmark will be a fully hedged bond benchmark, to which we add a long only currency overlay $a = 1$, $b = 0$. Empirically, we analyse a dollar bonds index (SSB all maturities) from two different bases, Euro (Deutschmark prior to 1999) and Yen, over the period end of May 1987 to July 2002. Table 3 indicates the monthly returns achieved by the active overlay programs on a stand alone basis, H as well as its correlation with the passive benchmark. Overall, the estimates implied from the joint distribution of (X,F,T) are close to direct observations (Table 3), especially the standard deviation of the overlay program as well as the correlation with the benchmark. Expected value derived from the model are lower than observed in the markets suggesting that assuming multivariate normal distributions can only be a first approximation and that further research is needed to modelize more closely the expected value of directional strategies.

Table 1: Underlying currency markets X,
Monthly statistics from end of May 1987 to end of July 2002

	$/Euro	$/Yen
Mean (X)	0.073%	0.151%
Stdev(X)	3.179%	3.657%

Table 2: Forecasting strategies F

	$/Euro		$/Yen	
	AR(1)	FRB	AR(1)	FRB
Mean (F)	0.073%	0.025%	0.151%	0.264%
Stdev(F)	3.179%	0.242%	3.657%	0.208%
Corr(F,X)	0.139	0.128	0.019	0.191

Table 3: Directional Overlay program H and combination with Benchmark T

	Dollar bond investments from a Euro base						Dollar bond investments from a Yen base					
	AR(1)			FRB			AR(1)			FRB		
	0.5 l/s	short	long	0.5 l/s	short	long	0.5 l/s	short	long	0.5 l/s	short	long
Mean (H)	0.289%	0.252%	0.325%	0.211%	0.175%	0.248%	0.243%	0.168%	0.319%	0.234%	0.159%	0.310%
From model	0.177%	0.141%	0.214%	0.165%	0.128%	0.201%	0.030%	-0.045%	0.106%	0.184%	0.109%	0.260%
Stdev(H)	1.563%	2.410%	2.032%	1.576%	2.017%	2.439%	1.814%	2.504%	2.645%	1.815%	1.094%	3.475%
From model	1.580%	2.218%	2.264%	1.581%	2.146%	2.334%	1.830%	2.543%	2.630%	1.821%	1.196%	3.448%
Mean(T)	0.675%	0.711%	0.638%	0.675%	0.711%	0.638%	0.474%	0.550%	0.398%	0.474%	0.550%	0.398%
Stdev(T)	1.871%	3.214%	1.315%	1.871%	3.214%	1.315%	2.139%	3.758%	1.308%	2.139%	3.758%	1.308%
Corr(T,X)	0.723	0.915	-0.180	0.723	0.915	-0.180	0.794	0.938	-0.100	0.794	0.938	-0.100
Corr(T,F)	0.052	0.099	-0.095	-0.024	0.049	-0.189	0.013	0.016	-0.006	0.066	0.131	-0.158
Corr(T,H)	-0.072	-0.685	-0.062	0.191	-0.562	-0.088	0.050	-0.645	-0.066	0.715	-0.231	-0.040
From model	0.014	-0.643	-0.130	0.060	-0.622	-0.134	0.027	-0.652	-0.072	0.629	-0.308	-0.090

Conclusion

Systematic programs are popular among commodity trading advisors and hedge fund managers. A better understanding of directional strategies can be achieved using stochastic modelling. The first two moments, expected value and variance are derived analytically for a general class of trading rules. The correlation coefficient between the active program and a tracker fund is subsequently derived allowing for mean-variance allocation. When the underlying markets shows a positive drift and the goal is to maximize the return to risk ratio on the investment, a portfolio including both a tracker fund and a long/short strategy will be superior to both investments considered separately. The forecasting model used to time the derivatives markets will have to take into account the tracker fund it is associated to if the goal is to provide the highest return to risk ratio. An avenue for further research would include the consideration of non-symmetrical directional strategies. Indeed in a market with positive drift, it could well be that an asymmetrical strategy outperforms the combination of tracker fund with an actively managed futures contracts. This will involve redefining the risk analytics and going beyond the traditional mean variance analysis which cannot capture the risk of dynamic strategies such as shorting options or take profits rules.

An empirical application to the currency markets suggest that the theoretical model explain both the standard deviation of returns generated by the active program as well as its correlation with a tracker fund. Expected values are underestimated in sample. These results indicate that assuming multivariate normal distributions can only be a first approximation and that further research is needed.

Appendix 1 **Mean and Variance of Directional Strategies**

$$H = \begin{cases} a\,X & \text{if} \quad F > 0 \\ b\,X & \text{if} \quad F < 0 \end{cases}$$

Let us note the trading signal $B = \begin{cases} a & \text{if} \quad F > 0 \\ b & \text{if} \quad F < 0 \end{cases}$

$$H = B\,X$$

$$\begin{bmatrix} X \\ F \end{bmatrix} \sim N(\begin{bmatrix} \mu_x \\ \mu_f \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \rho_{xf}\sigma_x\sigma_f \\ \rho_{xf}\sigma_x\sigma_f & \sigma_f^2 \end{bmatrix})$$

Let us note $X^* = (X - \mu_x)/\sigma_x$ and $F^* = (F - \mu_f)/\sigma_f$ the standard normalized variables.

$$H = B\,X = B(\mu_x + \sigma_x X^*) = \mu_x B + \sigma_x B\,X^*$$

$$E(H) = \mu_x E(B) + \sigma_x E(B\,X^*)$$

$$E(H) = \mu_x [a\ \Phi(\frac{\mu_f}{\sigma_f}) + b\ \Phi(-\frac{\mu_f}{\sigma_f})] + \sigma_x (\int_{X^*} \int_{F^* > -\frac{\mu_f}{\sigma_f}} a\,X^* + \int_{X^*} \int_{F^* < -\frac{\mu_f}{\sigma_f}} b\,X^*)$$

Using the results of Kotz, Balakrishnan and Johnson (2000: p311-2 as well as 315) on truncated bivariate distributions

$$E(H) = \mu_x [a\ \Phi(\frac{\mu_f}{\sigma_f}) + b\ \Phi(-\frac{\mu_f}{\sigma_f})] + \sigma_x (\frac{a}{\sqrt{2\pi}} \rho_{xf} \exp(-0.5\frac{\mu_f^2}{\sigma_f^2}) - \frac{b}{\sqrt{2\pi}} \rho_{xf} \exp(-0.5\frac{\mu_f^2}{\sigma_f^2}))$$

$$E(H) = \mu_x [a\ \Phi(\frac{\mu_f}{\sigma_f}) + b\ \Phi(-\frac{\mu_f}{\sigma_f})] + \sigma_x \frac{(a-b)}{\sqrt{2\pi}} \rho_{xf} \exp(-0.5\frac{\mu_f^2}{\sigma_f^2})$$

$$H^2 = \mu_x^2 B^2 + 2\mu_x\sigma_x B^2\,X^* + \sigma_x^2 B^2\,X^{*2}$$

$$E(H^2) = \mu_x^2 E(B^2) + 2\mu_x\sigma_x E(B^2\,X^*) + \sigma_x^2 E(B^2\,X^{*2})$$

$$E(B^2) = a^2\ \Phi(\frac{\mu_f}{\sigma_f}) + b^2\ \Phi(-\frac{\mu_f}{\sigma_f})$$

Using Kotz, Balakrishnan and Johnson (2000: p311)

$$E(B^2 X^*) = \frac{(a^2 - b^2)}{\sqrt{2\pi}} \rho_{xf} \exp(-0.5\frac{\mu_f^2}{\sigma_f^2})$$

Using Kotz, Balakrishnan and Johnson (2000: p312)

$$E(B^2\,X^{*2}) = a^2(\frac{\rho_{xf}^2}{\sqrt{2\pi}}(-\frac{\mu_f}{\sigma_f})\exp(-0.5\frac{\mu_f^2}{\sigma_f^2}) + \Phi(\frac{\mu_f}{\sigma_f})) + b^2(\frac{\rho_{xf}^2}{\sqrt{2\pi}}(\frac{\mu_f}{\sigma_f})\exp(-0.5\frac{\mu_f^2}{\sigma_f^2}) + \Phi(-\frac{\mu_f}{\sigma_f}))$$

Therefore:

$$E(H^2) = \mu_x^2 [a^2\ \Phi(\frac{\mu_f}{\sigma_f}) + b^2\ \Phi(-\frac{\mu_f}{\sigma_f})] + 2\mu_x\sigma_x \frac{(a^2 - b^2)}{\sqrt{2\pi}} \rho_{xf} \exp(-0.5\frac{\mu_f^2}{\sigma_f^2})$$

$$+ \sigma_x^2 [a^2(\frac{\rho_{xf}^2}{\sqrt{2\pi}}(-\frac{\mu_f}{\sigma_f})\exp(-0.5\frac{\mu_f^2}{\sigma_f^2}) + \Phi(\frac{\mu_f}{\sigma_f})) + b^2(\frac{\rho_{xf}^2}{\sqrt{2\pi}}(\frac{\mu_f}{\sigma_f})\exp(-0.5\frac{\mu_f^2}{\sigma_f^2}) + \Phi(-\frac{\mu_f}{\sigma_f}))]$$

Appendix 2 Correlation with Tracker Fund

Here, we explicitly assume that the joint distribution of the underlying returns X, the forecast F and the tracker fund T is a trivariate normal distribution denoted by:

$$\begin{bmatrix} X \\ F \\ T \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_x \\ \mu_f \\ \mu_t \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \rho_{xf}\sigma_x\sigma_f & \rho_{xt}\sigma_x\sigma_t \\ \rho_{xf}\sigma_x\sigma_f & \sigma_f^2 & \rho_{ft}\sigma_f\sigma_t \\ \rho_{xt}\sigma_x\sigma_t & \rho_{ft}\sigma_f\sigma_t & \sigma_t^2 \end{bmatrix}\right)$$

Let's note the portfolio P=T+H.

$$E(P) = \mu_t + E(H)$$

$E(P^2)$ can be developed using the argufies which were invoked to establish $E(H^2)$. Indeed, we only need to remark that:

$$P = \begin{Bmatrix} T + aX = Y & \text{if} & F > 0 \\ T + bX = Z & \text{if} & F < 0 \end{Bmatrix}$$

Therefore both [Y,F] and [Z,F] also follow bivariate normal distributions.

$$Y \sim N(\mu_t + a\mu_x, \sigma_t^2 + a^2\sigma_x^2 + 2a\rho_{tx}\sigma_t\sigma_x)$$

$$Cov(Y,F) = Cov(T,F) + a\,Cov(X,F)$$

$$Corr(Y,F) = \frac{\rho_{tf}\sigma_t + a\rho_{xf}\sigma_x}{\sqrt{\sigma_t^2 + a^2\sigma_x^2 + 2a\rho_{tx}\sigma_t\sigma_x}}$$

Similarly, $Z \sim N(\mu_t + b\mu_x, \sigma_t^2 + b^2\sigma_x^2 + 2b\rho_{tx}\sigma_t\sigma_x)$ and

$$Corr(Z,F) = \frac{\rho_{tf}\sigma_t + b\rho_{xf}\sigma_x}{\sqrt{\sigma_t^2 + b^2\sigma_x^2 + 2b\rho_{tx}\sigma_t\sigma_x}}$$

We know from Appendix 1, that by choosing a value of $a = 1$, $b = 0$ in Equation [3]:

$$E(H^2)\Big|_{a=1,b=0} = \int_X \int_{F>0} X^2 = \mu_x^2\Phi(\delta_f) + 2\mu_x\sigma_x \frac{\rho_{xf}}{\sqrt{2\pi}}\exp(-\frac{\delta_f^2}{2}) + \sigma_x^2(\frac{-\rho_{xf}^2}{\sqrt{2\pi}}\delta_f \exp(-\frac{\delta_f^2}{2}) + \Phi(\delta_f))$$

If we note $g[\mu_x, \sigma_x^2, \delta_f, \rho_{xf}]$ this function, we obtain:

$$E(P^2) = g[\mu_t + a\mu_x, \sigma_t^2 + a^2\sigma_x^2 + 2a\rho_{tx}\sigma_t\sigma_x, \delta_f, \frac{\rho_{tf}\sigma_t + a\rho_{xf}\sigma_x}{\sqrt{\sigma_t^2 + a^2\sigma_x^2 + 2a\rho_{tx}}}]$$

$$+ g[\mu_t + b\mu_x, \sigma_t^2 + b^2\sigma_x^2 + 2b\rho_{tx}\sigma_t\sigma_x, -\delta_f, -(\frac{\rho_{tf}\sigma_t + b\rho_{xf}\sigma_x}{\sqrt{\sigma_t^2 + b^2\sigma_x^2 + 2b\rho_{tx}}})]$$

$$Var(P) = E(P^2) - (E(P))^2$$

and $Corr(T,H) = \dfrac{Var(P) - \sigma_t^2 - Var(H)}{2\,\sigma_t\sqrt{Var(H)}}$

Appendix 3 Optimal Allocation

Optimal allocation between an index and the maximum return long/short futures fund

If we denote H^*_{max} the returns derived by the optimal predictor on the dritfless futures markets, we obtain from equation [2]:

$E(H^*_{max}) = \sigma_x \sqrt{\frac{2}{\pi}} \rho_{xf}$ and $Var(H^*_{max}) = \sigma_x^2 (1 - \frac{2}{\pi}\rho_{xf}^2)$

We also know from Section 2 that for long/short strategies in a driftless market $Cov(H^*_{max}, X^*) = Cov(H^*_{max}, X) = 0$.

The second step is to allocate the optimal weight w between tracker and hedge fund given by [5].

$$\tilde{w} = \frac{\sigma_x}{\mu_x} \frac{\sqrt{\frac{2}{\pi}} \rho_{xf}}{(1 - \frac{2}{\pi}\rho_{xf}^2)} = \frac{1}{\delta_x} \frac{\sqrt{\frac{2}{\pi}} \rho_{xf}}{(1 - \frac{2}{\pi}\rho_{xf}^2)}$$

As a consequence:

$$E(P) = \mu_x + \frac{1}{\delta_x} \frac{\frac{2}{\pi}\rho_{xf}^2 \sigma_x}{1 - \frac{2}{\pi}\rho_{xf}^2} = \sigma_x (\delta_x + \frac{1}{\delta_x} \frac{\frac{2}{\pi}\rho_{xf}^2}{1 - \frac{2}{\pi}\rho_{xf}^2})$$

$$E(P) = \mu_x + \sigma_x \frac{\frac{2}{\pi}\rho_{xf}^2}{1 - \frac{2}{\pi}\rho_{xf}^2} = \sigma_x (\frac{\delta_x^2 + \frac{2}{\pi}\rho_{xf}^2}{\delta_x (1 - \frac{2}{\pi}\rho_{xf}^2)})$$

$$Var(P) = \sigma_x^2 + \tilde{w}^2 (1 - \frac{2}{\pi}\rho_{xf}^2)\sigma_x^2 = \sigma_x^2 \, (1 + \frac{\frac{2}{\pi}\rho_{xf}^2}{\delta_x^2 (1 - \frac{2}{\pi}\rho_{xf}^2)})$$

$$Var(P) = \sigma_x^2 \, \frac{\delta_x^2 + \frac{2}{\pi}\rho_{xf}^2 (1 - \delta_x^2)}{\delta_x^2 (1 - \frac{2}{\pi}\rho_{xf}^2)}$$

$$SR(P) = \frac{\delta_x^2 + \frac{2}{\pi}\rho_{xf}^2}{\sqrt{1 - \frac{2}{\pi}\rho_{xf}^2} \sqrt{\delta_x^2 + \frac{2}{\pi}\rho_{xf}^2 (1 - \delta_x^2)}}$$

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