

When Normality is not enough

By: Peter Urbani

One of the simplifying assumptions of Modern Portfolio Theory is that returns are normally distributed.

This assumption was made by Harry Markowitz in 1952, when he chose to use variance as the measure of risk. Variance is calculated as the sum of the squared differences from the mean or average. More typically we take the square root of this number to get the standard deviation.

The normal, Guassian or Bell shaped distribution is a symmetrical distribution that can be fully described by its mean and standard deviation and therein as we will show lies its problem.

Markowitz's reasons for using variance were largely pragmatic and due to the lack of computing power at the time which required him to perform most of the necessary calculations by hand.

He later went on to say that semi-variance or downside deviation might be a more realistic representation of the way in which investors perceive risk. I.E. That they feel more pain from the same level of losses than the pleasure they get from the same level of gains.

In a more recent paper, Markowitz goes on to say that Modern Portfolio Theory is a 'normative' as opposed to 'positive' theory in that it describes how investors ought to behave rather than how they necessarily do behave.

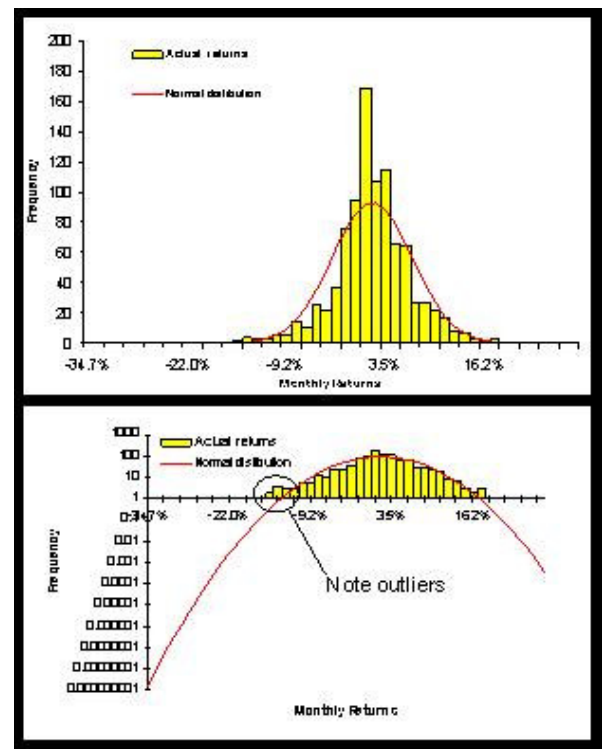
Most of the time the assumption of normality appears good enough to describe everyday events in the market place or the returns under normal circumstances. But the normal distribution has one major failing and that is that it drastically understates the size and frequency of extreme events.

This is because the ends of the normal distribution tail off too quickly to adequately describe the size and frequency of extreme events. Under the normal distribution, 68,3% of all observations should fall within 1 standard deviation of the mean, 95,4% within 2 standard deviations and 97,7% within 3 standard deviations. Thus under the normal distribution there is only a 0,3% probability of observing a greater than 3 standard deviation event.

If we consider the monthly returns from the JSE All Share Index since 1925 we get a monthly return of just over 1% and a monthly standard deviation of around 5%.

Under the assumption of normality this means that there is only a 0,3% probability of observing a monthly return of more than +16% or less than -14%. Out of the available 948 data points we should thus observe only about 3 periods when the returns were outside this range. In fact there have been 11 months when the monthly return on the JSE All Share index has been less than -14%. This is almost 9 times as often as predicted under the normal distribution.

This is not so evident when looking at the distribution of the JSE against the normal distribution on a normal scale. But when viewed on a log-scale the magnitude of the underestimation becomes more obvious.



The reason for these 'fat-tails' is that the JSE like many other financial markets exhibits *leptokurtosis* (Greek for 'Thin Arches'). This simply means that the actual distribution has excess kurtosis or is more peaked than the normal distribution. In addition the JSE also has negative skewness which explains the asymmetry of returns (more downside extremes than upside extremes).

In the case of the JSE and indeed most markets the mean and standard deviation alone are not enough to fully describe the shape of the distribution of returns. We need to also take the higher moments of skewness and kurtosis into account.

There are a number of ways of doing so. For the purposes of this article I shall describe four such methods and compare the Value at Risk (VaR) and Expected Shortfall (ES) measures for each.

The first such 'fat tailed' distribution we could use is the student-t distribution. In fact its use is recommended whenever we are dealing with a sample rather than the full population of observations.

The second method is match the moments of the distribution using a mixture of normal distributions.

The third is to modify the normal distribution to take account of the excess skewness and kurtosis using the Edgeworth expansion.

The fourth method is to use an extreme value type distribution. Here we use the Generalised Pareto or Gumbel distribution.

We use the annualised statistics of the JSE All Share index from 1925 – 2003 from the Firer and McLeod database. This gives the following Mean, Standard Deviation, Skewness and Kurtosis.

JSE All Share Index (1925 - 2003)	
Mean	13.13%
Std Deviation	17.87%
Skewness	-0.65
Kurtosis	4.57
Degrees of freedom	3
Confidence level	0.99
Number of observations	948

We calculate the parametric Value at Risk and Expected Shortfall using the four different distributions and then compare the number of exceedences in the actual distribution to get the following results.

Value at Risk (VaR)	VaR	No. Obs below	%
VaR Normal @ 99%	-28.44%	39	4.11%
VaR Student T @ 99%	-33.72%	25	2.64%
VaR Vol Mix @ 99%	-41.61%	16	1.69%
VaR Gumbel @ 99%	-42.93%	16	1.69%
VaR Cornish Fisher @ 99%	-48.24%	12	1.27%

Remember the Value at Risk at the 99% is that *minimum* level of loss that we can expect 1% of the time. Given 948 observations we would expect to have about 10 observations in excess of this amount.

The Expected Shortfall is the *average* loss that is realised when the VaR is exceeded. It is also called the conditional value at risk or extreme tail loss (ETL).

Expected Shortfall	ETL	No. Obs below	%
ETL Normal @ 99%	-34.50%	25	2.64%
ETL Student T @ 99%	-57.12%	5	0.53%
ETL Vol Mix @ 99%	-55.16%	7	0.74%
ETL Gumbel @ 99%	-56.45%	6	0.63%
ETL Cornish Fisher @ 99%	-65.41%	3	0.32%

As can be seen, both the Value at Risk and Expected Shortfall, calculated using the assumption of normality, significantly underestimate the number and size of tail or extreme events. Fortunately for the purposes of this article we have used the 99% confidence level which means that annual losses of up to 70% should only be experienced 1 in every 100 years. At the 95% or once in every 20 years investors should expect a loss of up to 40%, still almost double that suggested by the normal distribution.

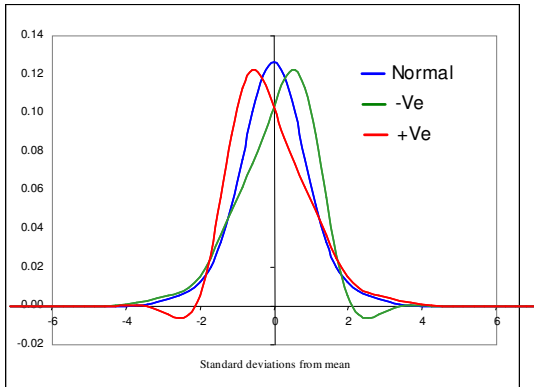
It is disturbing to the author that Hedge Funds are typically marketed on the strength of their lower than market risk as evidenced by their low standard deviations, when in fact Hedge Funds are particularly poor candidates for having their risk measures using a normal measure. This is because Hedge Funds are supposed to have positive skewness (more up months than down months) by design. Thus any symmetrical measure of risk used for a Hedge Fund risks under- or over-stating its true level of risk.

In conclusion then investors should be wary of marketing hype and always remember the sting in the tail. They should take note of higher moments, particularly negative skewness and excess kurtosis and should demand to see downside risk measures for all investments.

References:

The Legacy of Modern Portfolio Theory, Frank J Fabozzi, Francis Gupta and Harry Markowitz, Institutional Investor Magazine

Why Skewness Matters



	A	B	C
Mean	0.0	0.0	0.0
Var	1.0	1.0	1.0
SD	1.0	1.0	1.0
Skew	1.0	0.0	-1.0
Kurt	3.6	3.9	3.6

CL 0.99

Value at Risk (VaR) for a portfolio of RIm

Assuming Normality

	-R 23,154	-R 23,263	-R 23,020
	-2.32%	-2.33%	-2.30%

After adjusting for Skewness & Kurtosis

	-R 23,165	-R 25,297	-R 37,053
VaR	-2.32%	-2.53%	-3.71%
ETL	-2.64%	-3.09%	-4.60%

In the above example of three hypothetical funds, all three have the same mean and standard deviation. However fund **A** is positively skewed while fund **C** is negatively skewed.

If the Value at Risk for each of these three investments is calculated under the assumption of normality using just the means and standard deviation, then the downside risk of the three funds is identical save for rounding errors.

However, if we take the excess skewness and kurtosis into account by using the modified Value at Risk formula, it becomes clear that fund **C** has significantly more downside risk than either **A** or **B**. This is because of the negative skewness of its distribution relative to that of the other two funds. As you move further out into the tail of the distribution the degree of difference becomes even more marked as evidenced by the Extreme Tail Loss (ETL), also known as Expected Shortfall and Conditional Value at Risk, of fund **C**. The JSE All Share index has long-run skewness and kurtosis of -0.65 and 4.57 respectively.

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