

The Omega Risk Measure

By: Peter Urbani

One of the main criticisms of risk measures is that they tend to concentrate solely on losses or the potential for losses and say nothing about how an investor achieves their other objective which is to achieve a specific target rate-of-return or higher.

Although it has been demonstrated that the minimum variance portfolio, invariably outperforms in the long-run, most people adopt Keynes's "In the long-run we are all dead" attitude to this and are not satisfied with reducing downside risk alone. They would also like to enhance their Alpha or excess return.

In addition, the increasing use of options, futures and hedge funds in portfolios is ensuring that the probability distributions of some funds are far from normal. How then can we assess the relative risks of these funds?

Sometimes the simplest ideas are the best. There is now widespread agreement that the 'best' downside risk measure is the probability weighted value of average returns *below* a certain threshold. This is known variously as the Expected Shortfall, conditional value at risk and Extreme Tail loss measure. If that methodology gives the 'best' downside risk number then surely the best overall measure of risk-adjusted relative returns must be the ratio of the probability weighted value of excess returns above a given threshold to the expected shortfall measure. In essence, the ratio of Gains to Losses.

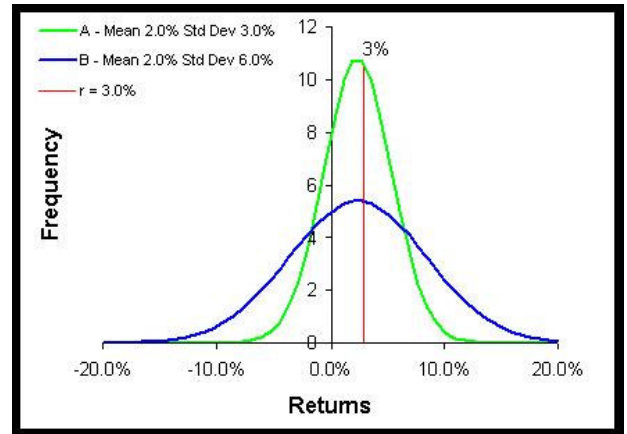
Such upside potential ratio's, are not new (see Sortino and Satchell), but hitherto have tended to be calculated by parametric means. This requires an assumption to be made about the shape of the distribution in question (normal, lognormal, 3-param etc). The key insight of the Shadwick and Keating, Omega measure, is not to make any assumption about the shape of the distribution, but rather to let the historical data speak for itself. Obviously the usual caveats about the shape of the future distribution not necessarily being the same as those of the past, applies. But lets face it what else do we have to go on?

Another main advantage of using the actual distribution is that all the additional moments beyond the skewness and kurtosis are also considered. This makes the Omega measure suitable for assessing even the most asymmetric, bi-modal and fat-tailed distributions such as those typically exhibited by hedge funds.

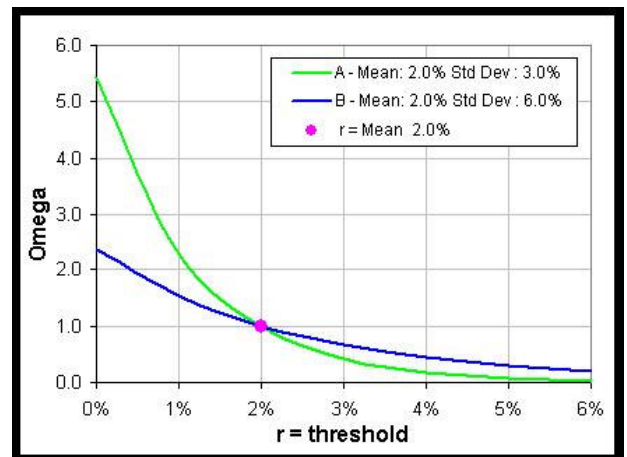
The other insight of William Shadwick and Con Keating is that the Omega measure should ideally not be used as a point estimate as with Sharpe, Sortino and other ratios, but that the entire shape of

the distribution with respect to a range of threshold returns should be considered.

The reason for this is simple. If you are both risk averse and greedy, as most of us are, then there is a certain point at which you should / have to accept more risk to get more return. The Omega measure tells you what this point is relative to a benchmark or alternative investment.



If we consider the above two normal fund distributions. Both have a mean return of 2% but one has a standard deviation of 6% (B) whilst the other is only 3% (A). In isolation we would always prefer A to B, but if we had to earn a minimum rate of return of say 3% then B is suddenly a better option because it has a higher probability of delivering returns above the threshold.



Moreover, if we plot the Omega measure with respect to a range of target returns we can see that B dominates A for all thresholds greater than 2%. In addition, we can see that the Omega function returns a value of 1 for a threshold r of 2%. This is the mean return for both funds and corresponds to the level at which the Gains / Losses = 1. Given the choice between two normally distributed funds with identical means but different variances, the Sharpe ratio will always choose the fund with the lowest variance in this case A.

At a given return threshold using the simple rule of preferring more to less an asset with a higher Omega is preferable to one with a lower Omega.

The Omega measure is able to handle even the most extreme asymmetry as in the lottery problem. Consider whether it is better to buy or sell a single lottery ticket with the following payoff:

Buy payoff

- a loss of 90cents 999,999 times in 1 million
- a gain of 999,999 one time in 1 million.

Sell payoff

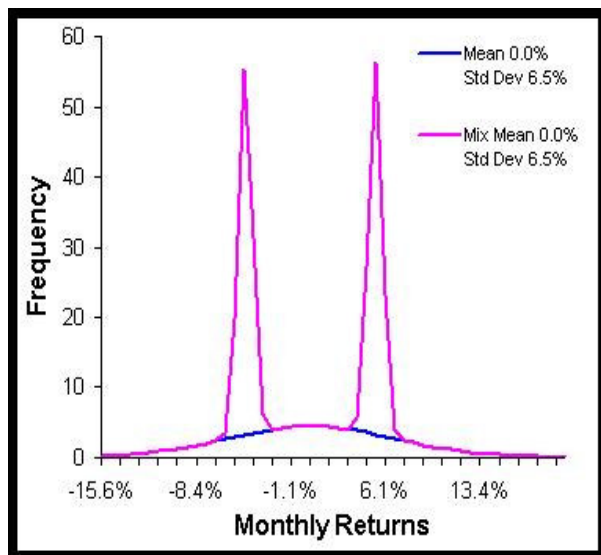
- a gain of 90cents 999,999 times in 1 million
- a loss of 999,999 one time in 1 million.

Both have a mean of 90c and a variance of 999,999.

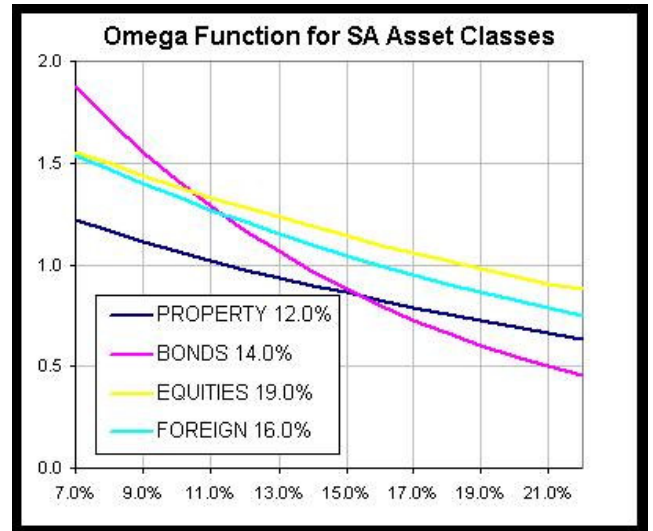
We all know that it is preferable to buy the lottery ticket than to sell it (unless you are lthingo with a licence and a reasonable chance of selling millions of tickets), but traditional measures such as the Sharpe ratio cannot distinguish between the two options.

This leaves the Sortino ratio. It correctly identifies the heavy downside bias of selling a lottery ticket and chooses to Buy one for MAR below the mean. However, it fails the lottery test by reversing its choice for MAR above the mean. The Omega measure correctly identifies Buying the lottery ticket throughout.

The Sharpe and Sortino ratios are also unable to differentiate between multi-modal distributions such as the one shown below.

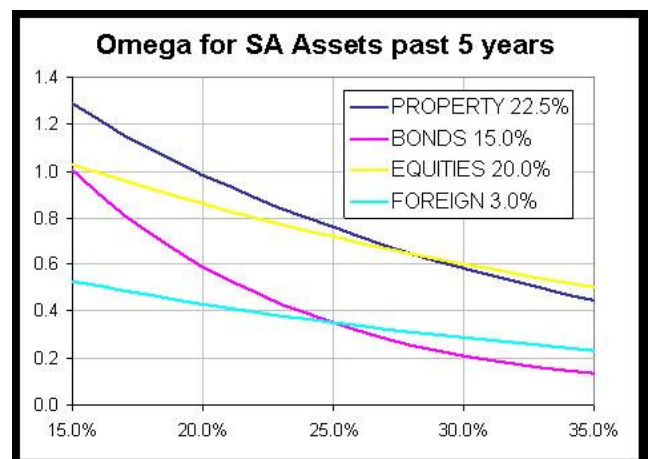


If we now turn our attention to how the Omega measure might be used in practice, by looking at the full distribution for a range of threshold returns for various asset classes, we can see a number of interesting things.



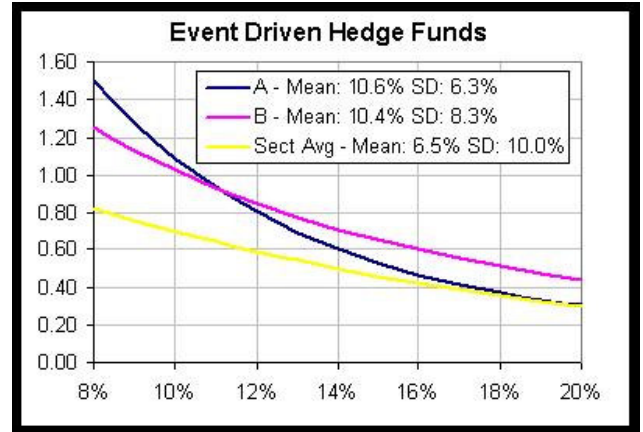
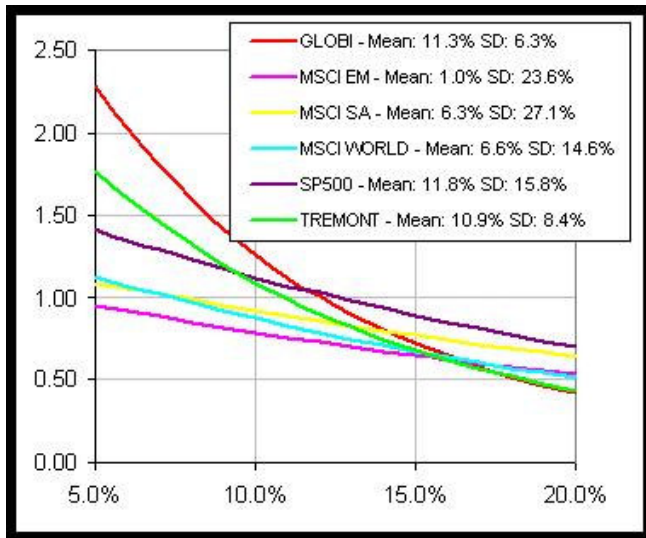
If we look at the Omega measure for SA Asset Classes since 1977 we can see that property, as denoted by the property unit trust index, has never out-performed equities although it does do better than bonds at return thresholds of above 15%.

This is at odds with what we have experienced over the past 5 years and investors would do well to question what has changed. The obvious answer being the dramatic fall in interest rates from a high of 28% to the current 7,5%. Anytime inflation halves and the cost of capital falls by more than 2/3rds, income generating assets are likely to do well. The question now is what happens next if and when interest rates start to rise again.



As you can see from the above chart Property has been by far the best performing asset class over the past 5 years with equities only out-performing at thresholds of around 30%.

Internationally we can see a similar picture with global bonds, as denoted by the JP Morgan global bond index, having been the best performing asset class as a result of the disinflationary pressures stemming from Asia. The second best performing asset class has been hedge funds as denoted by the Credit Suisse First Boston (CSFB) Tremont hedge fund index. It is interesting to note that the S&P500 index gave a superior probability weighted return to hedge funds at threshold levels of above 9% and beat global bonds at required returns levels of 12% and above.



Whilst both funds strongly outperformed the benchmark, Fund B had a greater likelihood of delivering more than 11% and also a more consistent tracking error to the benchmark.

$$\Omega = \frac{1 - F(r)}{F(r)} \times \frac{\bar{x}(r, U) - r}{r - \bar{x}(L, r)}$$

\bar{x} = the Mean Return (from L to U)
 F (r) = the probability that a return is below r
 $\bar{x}(L, r)$ = average returns below r
 1 - F (r) = the probability that a return is above r

| | Mean | SD | SD | DD | ES | VaR | Sharpe | Sortino | 1% | 10% | 15% |
|------------|-------|-------|----|----|----|-----|--------|---------|-------|-------|-------|
| | | | | | | | | | Omega | Omega | Omega |
| GLOBI | 11.3% | 6.3% | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 3 |
| MSCI EM | 1.0% | 23.6% | 6 | 6 | 6 | 6 | 7 | 7 | 6 | 6 | 6 |
| MSCI SA | 6.3% | 27.1% | 7 | 7 | 7 | 7 | 5 | 5 | 5 | 4 | 2 |
| MSCI WORLD | 6.6% | 14.6% | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 5 | 5 |
| SP500 | 11.8% | 15.8% | 5 | 5 | 5 | 5 | 3 | 3 | 3 | 2 | 1 |
| TREMONT | 10.9% | 8.4% | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 3 | 4 |
| USTB3M | 4.2% | 0.5% | 1 | 1 | 1 | 1 | 6 | 6 | 7 | 7 | 7 |

If we compare the relative rankings of these asset classes using different risk measures we can see that downside risk measures are fairly consistent, only the Omega measure is able to change the ranking at different thresholds. Thus for instance it correctly downgrades the ranking of global bonds to 3rd at required return levels of 15% or more as both the S&P500 and MSCI South Africa indices have a higher probability of exceeding this threshold by virtue of their higher volatilities.

The Omega measure can also be used to differentiate between funds and to assess the skill level of the manager to the benchmark as in the case of the two apparently identical hedge funds shown in the next chart.

$$\Omega = \text{Risk/Reward} = (rf - \bar{x}(L, rf)) / (\bar{x}(rf, U) - rf)$$

$$\Omega = (1/F(r) - 1) / \Omega_{\text{Benchmark}}$$

where r is the threshold return (which may be taken as a Risk-free Rate).

$$\text{Sharpe Ratio} = (\bar{x} - rf) / \sigma$$

$$\text{Sortino Ratio} = (\bar{x} - rf) / \text{DD}$$

$$\text{Sharpe - Omega} = \frac{\text{Expected return} - \text{threshold } r}{\text{Put Option price}}$$

$$\Omega = \frac{\text{Call Option price with strike } r}{\text{Put Option price with strike } r}$$

In conclusion then, the Omega measure offers a flexible, easy to use alternative to both the Sharpe and Sortino ratio's. Its non-parametric nature greatly reduces the potential for error and makes the ratio as statistically significant as the data it is assessing. It correctly deals with a number of Allais paradoxes related to invariance such as the lottery problem. When plotted on a log scale it can differentiate between even the most extreme asymmetries. As a probability weighted measure that considers both the upside and the downside it is ideally suited for use with hedge funds.

References:

A Universal Performance Measure – Keating and Shadwick (2002)

An Introduction to Omega – Keating and Shadwick (2002)

Omega as a performance measure – Kazemi, Scheeweis and Gupta (2003)

Optimal Asset Allocation with Omega – Avouyi-Dovi, Morin and Neto (2004)

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